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# ABSTRACT

The definition of geoidal undulations is given and after summarising the methods of determination of the geoid, computed geoidal undulations by some of the methods for several points in Kenya are compared to the results obtained by the satellite gravimetric solutions.

Results from astrogeodetic levelling and satellite altimetry show some reasonable agreement with the satellite-gravimetric geoids while results by Doppler satellite positioning indicate that good agreement can be obtained if the orthometric heights for the points are adjusted to a uniform system.

# INTRODUCTION

This paper gives the geoidal undulations in Kenya computed for the whole country using gravimetric and satellite derived solutions. The gravimetric solution is that of Gachari and Olliver, 1986; and the satellite solution is based on the GEM 10C earth model. Geoidal undulations have been computed at discrete points using results of Doppler satellite positioning and spirit levelling. Geoidal undulations differences have also been computed by astrogeodetic levelling at a few points. A comparison of the results for the discrete points is made with the values estimated from the geoidal maps of the gravimetric and satellite solutions. Comparison is also made for the satellite altimetry results for a few points on the coast.

# THEORETICAL CONSIDERATIONS

Definition: The geoid is the equipotential surface of the earth's attraction and rotation that best approximates the mean sea-level over the whole earth. The term was introduced by K.F. GAUSS in 1884 as the mathematical figure of the earth and as such it is a key figure in Geodesy, playing a fundamental role in positioning.

## Approximations of the geoid

- (i) Up to an accuracy of a few metres ( $\pm 2m$ ) the geoid is represented by the mean sea-level.
- (ii) Up to an accuracy of some tens of metres the geoid is represented by a biaxial geocentric ellipsoid whose minor axis coincides with the earth's principal polar axis of inertia. The biaxial ellipsoid is an analytically defined 'normal body of the earth' that best fits the geoid and is often referred to as the 'reference ellipsoid'.

Geoidal height - This is the undulation of the geoid obtained as the separation between the geoid and the reference ellipsoid.

## Causes of geoidal undulations:

Geoidal undulations are generally caused by the inaccurate approximation of the geoid by the reference ellipsoid because:

1. From the definition, geoidal undulations are brought about by the differences between the normal gravity field and the actual gravity field of the earth.
2. Where there are irregularities in the mass distribution, the geoidal undulations will be more pronounced even if the best fitting ellipsoid were adopted.

3. With a reference ellipsoid of dimensions (a,f) adopted, its positioning with respect to the geoid will also give rise to geoidal undulations if the positioning is not accurately done.

#### METHODS OF DETERMINATION OF GEOIDAL UNDULATIONS

The commonly used methods are:

1. Astronomical Levelling: This is suited for a local or regional area. The data required are the astrogeodetic deflections of the vertical. The method will give an accuracy of about  $\pm 4\text{m}$ . However the accuracy is not homogeneous and can be very much affected in rough topography.
2. Gravimetric determinations: Geoidal undulations at discrete points can be obtained by use of Stoke's integral. This method is suited for the whole earth but practically suited for limited areas of about  $1000\text{km} \times 1000\text{km}$ . It requires a dense gravity coverage and can give an accuracy of about  $\pm 1\text{m}$ .
3. Satellite fixes, e.g. Doppler Positioning - Geoidal undulations at discrete point at which the orthometric heights (H) are known can be determined from  $N=h-H$ , with h obtained as the geodetic height from the Doppler positioning. The same principle used on oceans, (in Satellite Altimetry) will give the difference between sea-level height and sea-surface height.
4. Potential coefficients - Potential coefficients together with dynamic form factor uniquely specify the normal gravity field. The coefficients are obtained from the analysis of perturbations of the orbits of several satellites. With the potential coefficients known, other parameters of the gravity field can be determined and hence the disturbing potential and finally by Bruns' formula, the geoidal undulations can be obtained. The accuracy by this method is about  $\pm 1\text{m}$ . It gives a globally homogeneous solution, but somewhat not detailed enough.
5. Other methods - Other methods used in geoidal determination are combinations of various data, usually done so as to take advantage of the effectiveness of the various methods as far as homogeneity, accuracy and detail coverage are concerned.

#### COMPUTED GEOIDAL UNDULATIONS

- (i) Gravimetric and satellite geoids: Fig. 1 shows a gravimetric geoid of Kenya computed on the GRS80 ellipsoid computed (by Gachari & Olliver, 1986) using a combination of GEM10B satellite derived potential coefficients and terrestrial gravity data. Fig. 2 shows the satellite derived geoid computed on the WGS72 ellipsoid, using potential coefficients based on GEM10C.
- (ii) Geoidal undulations from Doppler positioning - Geoidal undulations have been computed at some stations that are fixed by Doppler positioning. Most of the stations are part of the African Doppler Survey (ADOS) program and all are part of the Kenya geodetic network. With the heights of these points from the vertical control, the geoidal undulations on WGS72 ellipsoid are computed at these stations and shown in Fig. 1 and 2.
- (iii) Geoidal undulations from Astrogeodetic levelling - Astronomical levelling was computed at a few astrodeflection points shown in Fig 1 and 2 as A,B,C,...I. The differences in geoidal undulations,  $\Delta\text{NAL}$ , computed on the Clarke 1880 ellipsoid and converted to WGS72 ellipsoid are shown in the table below, alongside the estimated

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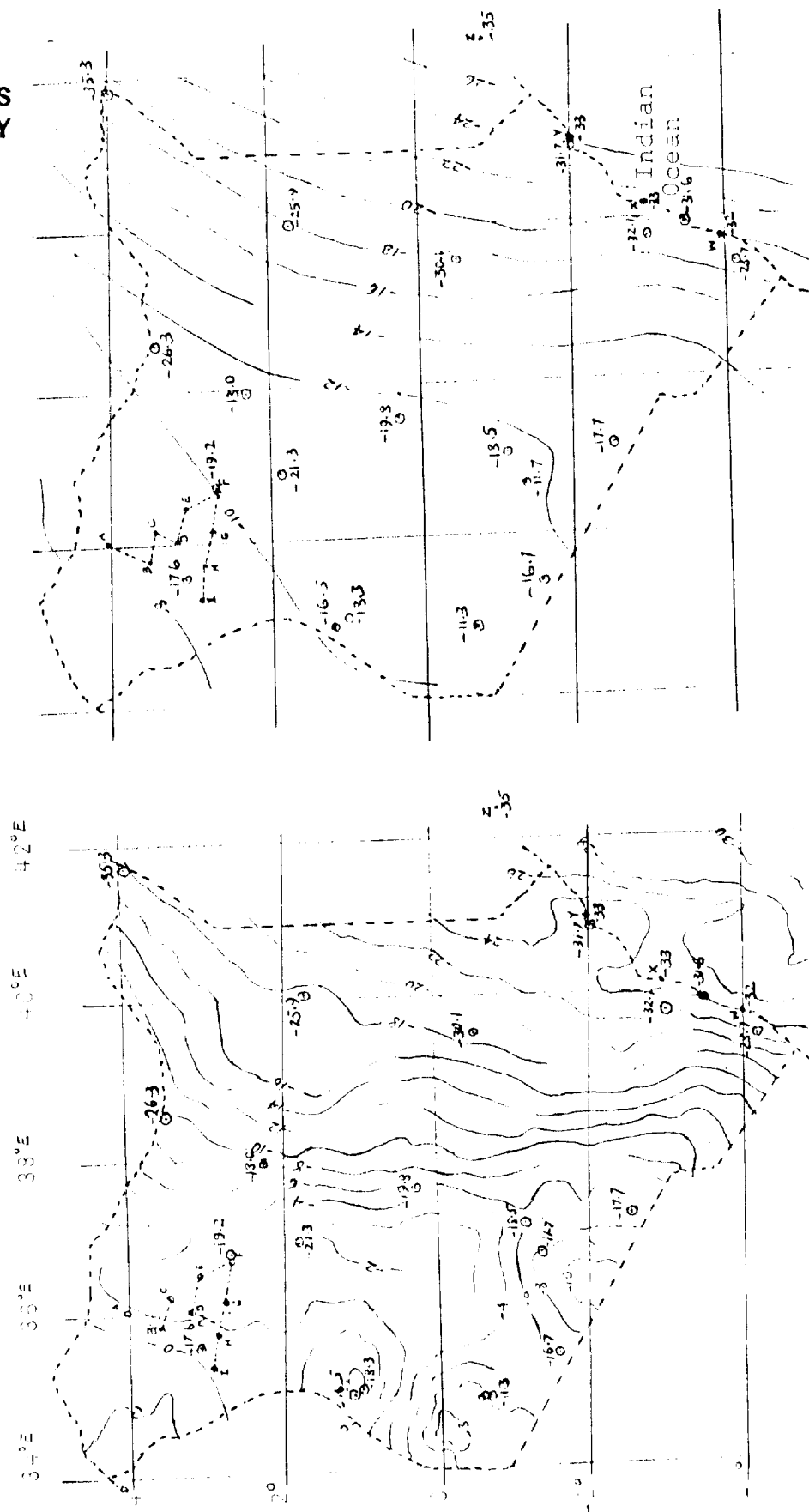


Fig. 2. Satellite derived geoid on 2m interval

Fig. 1. Gravimetric geoid of Kenya on 2m interval

differences in geoidal undulations from the geoidal maps for the gravimetric ( $\Delta N_G$ ) and satellite ( $\Delta N_{SD}$ ) geoids.

Line	$\Delta N_{AL}$	$\Delta N_G$	$\Delta N_{SD}$	Line	$\Delta N_{AL}$	$\Delta N_G$	$\Delta N_{SD}$
A-B	0.91m	-0.6m	0.0m	E-F	0.53m	-0.5	-0.6
B-C	-1.13	-0.4	-0.7	F-G	-0.79	-0.1	0.5
C-D	0.28	-0.6	0.0	G-H	0.73	-0.4	0.5
D-E	0.05	-0.8	-0.3	H-I	0.91	0.8	0.7

Table 1: undulations differences by various methods.

- (iv) Geoidal undulations from satellite altimetry - For the four points on the coast line, W,X,Y,Z, the values on GRS80 ellipsoid estimated from Rapp, 1982 are shown in Figs. 1 and 2.

#### COMPARISON OF RESULTS

The gravimetric and satellite derived geoids are similar in shape, particularly for the south-eastern and north-eastern parts of Kenya. However, the two differ in shape and detail as we go from the central part to the west and north-western. The differences in detail show up since satellite geoid is more generalised than the gravimetric geoid, while the similarity in shape is expected since the gravimetric geoid was computed incorporating potential coefficients. The differences in values could be due to the different ellipsoids and the gravity anomalies that were not corrected for terrain, indirect effect and atmospheric effects.

The Doppler derived undulations, for the points considered, differ from both the gravimetric and satellite derived geoids by mean values of about -12.7m and -8.8m respectively. These differences arise mostly due to errors in the orthometric heights. It is regrettable that the orthometric heights used to derive the Doppler geoidal undulations are not accurately computed as the vertical network was poorly observed and computed piecemeal.

The altimeter geoidal undulations estimated for the coastal shore points differ by about -8.3m and -10.1m from the estimated undulations of the gravimetric and satellite derived geoids respectively.

The comparison with astronomical levelling is relative as none of the points used has a fixed (known) geoidal undulation determined astrogeodetically. However, with mean differences of 0.5m and 0.2m for the relative geoidal height differences when compared with the gravimetric and satellite derived geoids respectively, it shows good agreement for the astronomical levelling method.

#### CONCLUSION

The gravimetric geoid has good agreement with the satellite derived geoid for the most parts of the country except for most of the western half of the country. This is likely to be due to the topography in the western half - it is mostly rugged and mountainous and in some parts rising to over 4000m above sea-level.

The Doppler derived undulations show some consistency with either the gravimetric or satellite derived geoid. If the orthometric heights can be accurately determined, these can improve on the undulations by Doppler positioning.

Astrogeodetic levelling can also be used to give more information for the geoidal undulations.

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